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How and Why Archery World Bow Test are Conducted

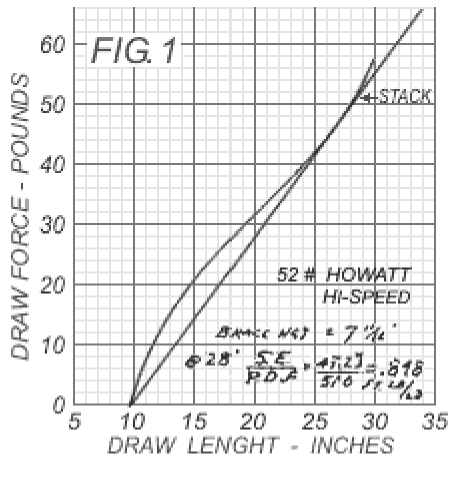
If you read this a bunch of times, there's a good chance you'll understand it. If not, don't worry; you're in company with the editors. However, we've had a lot of requests for the background on how our bow tests are conducted and calculated. We hope this answers those requests. And really, if you sit down with this material and a couple of our printed bow reports, you'll catch on. If you have the equipment to run your own tests, this information will be invaluable.

by Norb Mullaney

Bow tests have been performed for many years; they have ranged in quality from very bad to very good. Underlying all of this testing is a basic desire of archers to know certain things about the bow that are not readily apparent from casual shooting.

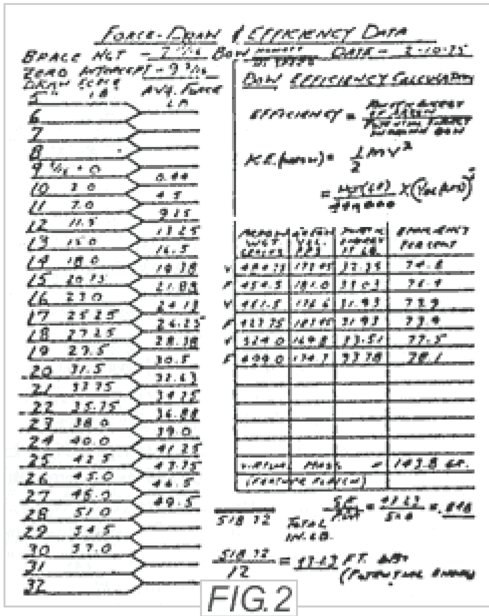
First, we want to know how much energy a bow stores when it is drawn and how efficiently it stores this energy. To measure this efficiency we should relate the energy stored to the maximum force that we must put forth when drawing the bow. After all, we have a limit as to the pull that we can exert and we want to shoot a bow we can handle. Within the limits of the pull we can muster, we'd like to store the greatest amount of energy possible.

To determine the total energy that is exerted (and stored) when the bow is drawn, we measure in one-inch intervals the force necessary to pull the string from brace height position to full draw. This information, when plotted in chart form, gives the force-draw curve for the bow. (See *Archery World*, October / November 1974 issue – "What We Can Learn From A Force-Draw Curve.")



For comparative bow tests we standardize all data by using the AMO standard 28-inch draw length as the value for testing. Consequently, the area under the force-draw curve represents the inch-pounds of energy that were required to draw the bow. This is the *potential or stored energy* that is available to restore the bow and propel the arrow when the string is released.

We calculate the stored energy by computing an average value of force for each one-inch segment of the force-draw curve and then adding all of the average values to obtain the area under the curve. Refer to Figure 2.



It is more usual and convenient to express the stored energy in foot-pounds rather than inch-pounds, so we divide by 12 to convert:

518.72 in.-lbs. / 12 = 43.23 ft.-lbs.
(Result is rounded to two decimal places.)

Thus, when the bow is drawn to 28 inches it has stored 43.23 ft.-lbs. of energy. We will call this value *stored energy* (E_s).

Since this is a conventional recurve bow, not a compound, the highest draw force obtained (peak draw force) was measured at the highest draw length (28 inches). When selecting a bow, we are interested in obtaining the maximum stored energy possible for a given peak draw force; we would like to have a measure of how efficiently the bow stores energy for comparative purposes. This measure of energy storing efficiency is obtained in terms of foot-pounds of stored energy per pound of peak draw force. It is calculated by dividing the stored energy by the peak drew force:

$$\frac{\text{Stored Energy}}{\text{Peak Draw Force}} = \frac{43.23\ \text{ft.-lbs.}}{51\ \text{lbs.}}$$

$$= 0.848\ \text{ft.-lbs./lb.}$$

At this point the static evaluation of the bow is complete. We may make adjustments to the bow or setup such as changing brace height or, on a compound, varying one or more of the many adjustable settings and then repeat the foregoing procedure. This is an excellent method of fine tuning a bow for optimum performance, or to obtain specific desirable characteristics to suit a personal requirement.

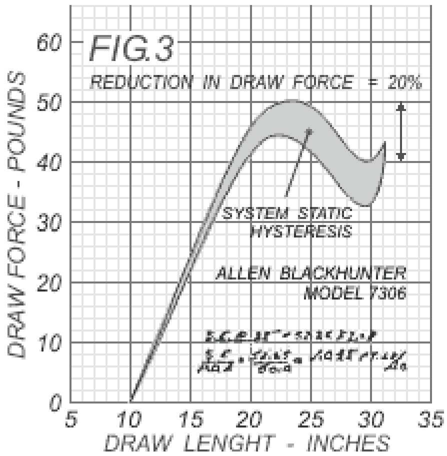
We have added one further element for compound bows and that is a determination of the *static hysteresis* for the system. As in any mechanism (and the bow is a mechanism) there is energy loss to friction within the system even in the simple bending and relaxing of the limbs. Under the dynamic conditions of a shot, energy is lost within the bow to friction and to air resistance, sound, heat and recoil. Some of these energy losses are very small, interrelated and quite complex, but they occur nevertheless.

(For straight or recurved composite bows using modern material and construction, exceptionally sensitive laboratory-type measuring equipment is required to register static hysteresis, and for practical purposes can be neglected.)

Hysteresis is the term applied to the energy lost in the system and hence not recoverable. It can be readily visualized in the following manner.

When a weighing scale is fastened to the bow string and the draw-force is measured and recorded for each one-inch increment as the string is moved from brace height to full draw, the resulting plot is the force-draw curve. If, as the bow is relaxed from full draw, new measurements of force are made at each one-inch position as the draw is relaxed from full draw to brace height, lower force-values will be obtained at each position. On self-bows (only one material used in construction – all wood, all fiberglass) the difference is readily discernible: however, on modern composite bows, the commercial type weighing scales normally used are not sensitive enough to read the difference. Assuming that the difference is sufficiently finite to be plotted on the force-draw curve, there will be a lower curve for the relaxing condition and a loop will be formed. This is known as hysteresis loop. The area within the loop is the hysteresis loss to the system or mechanism.

Now if we consider a compound bow with the eccentrics, idlers, cables and hangers, the loss of energy in friction within the system is very significant. The force-draw curve and hysteresis loop for a typical compound bow are shown in Fig.3. Note that the difference in the force values at the peak o the curve is about six pounds.



The term *static hysteresis* is used to denote this energy loss because the system is at rest when each measurement is made. In this manner the frictional forces are due to static friction. Friction of motion or kinetic friction under these conditions is generally less than static friction. Hence, if it were possible to measure the hysteresis while the bow was being shot, the hysteresis loop or loss would be substantially smaller.

Static hysteresis is only valuable as a characteristic for comparison between bows. Since it does not exist as measured under the dynamic conditions of shooting, it must be viewed in proper perspective. It is logical from an engineering standpoint to assume that the bow with the highest static hysteresis will also have the highest kinetic hysteresis and therefore the lowest efficiency, all other conditions being equal.

The energy lost to static hysteresis may be calculated by computing the area under the relaxing curve of the force-draw diagram (the section that forms the bottom side of the hysteresis loop) and subtracting this value from the stored energy. This gives the foot-pounds of energy within the loop and is the summation of system static hysteresis. It will be interesting to attempt to relate static hysteresis to bow efficiency as sufficient data becomes available to permit trend analysis.

The second, and probably the most important, characteristic we want to evaluate in a bow is *shooting performance*. A bow can store energy quite efficiently and yet not deliver an arrow very well.

Not all of the energy stored in thee drawn bow is imparted to the arrow when the shot is made. Some percentage of the stored energy is lost in the process of shooting the bow. It is dissipated in the recovery of the limbs and strings, there is minor loss to air resistance, heath and sound and also energy absorbed in recoil.

In a compound bow a significant amount of energy is consumed by friction in the eccentrics, idlers and cables of the compounding system. In addition, energy is required to overcome the translational and/or rotational inertia of the eccentrics, idlers and associated hardware required for the system.

We know, therefore, that the energy possessed by the moving arrow as it leaves the bow is some fraction of the total energy that we stored in the limbs when drawing the bow.

The laws of physics give the familiar formula for *kinetic energy* (energy of motion) of a moving body at any instant.

Formula 1)

kinetic energy $= E_k = ½\ m V^2$

where

rn = mass of the body in slugs
V = velocity of the body in feet per second
E_k = kinetic energy is in foot-pounds.

For those not familiar with the term "slug." it is the British absolute engineering unit of mass and is obtained by dividing the weight (in pounds) by the acceleration of gravity (32.16 ft/sec^2). Since the weight of an arrow is usually obtained in grains (1 lb. = 7000 grains), by substitution we can express the formula for the kinetic energy of the arrow in a more convenient form:

Formula 2)

E_k (ft.-lb.) $= W_a V^2 / 450240$

where

W_a = weight of the arrow in grains
V = arrow velocity in feet per second

PROBLEM: OBTAINING VELOCITY

We are all familiar with methods of weighing arrows on commercially available scales that read in grains. There remains the problem of obtaining the velocity of the arrow as it leaves the bow. This can be accomplished in a number of ways, but the modern approach is through the use of an electronic timer or chronograph in conjunction with a pair of trigger circuits. The trigger circuits, one to start the timer and one to stop the timer, are set with a predetermined distance between them so the arrow may be timed over a fixed distance.

The Kaufman chronograph we use has a 100.000 Hertz (cycles per second) oscillator with an octal readout. A chart is provided that converts the octal readout to velocity in feet per second. Two types of trigger circuits are provided: 1) a conductive grid printed on paper strips which triggers the timer when the arrow breaks the grid circuit; 2) a photoelectric circuit which is actuated when the arrow interrupts a beam of light. We prefer the photoelectric method because it can be reset with the touch of a button and creates no debris.

Knowing the arrow weight, the velocity is the simple matter to count the kinetic energy of the arrow in the Formula 2.

$E_k = W_a V^2 / 450240$
$E_k = 484.75 \times (173.45)^2 / 450240$
E_k = 32,39 ft.-lbs.
W_a = 484,75 grains
V = 173.45 feet per second

OVERALL EFFICIENCY DEFINED

While the kinetic energy of the arrow is an interesting value in itself, the principal use is in the computation of overall efficiency of the bow.

The overall efficiency of the bow is defined as the ratio of the kinetic energy of the arrow as it leaves the bow to the energy used to draw the bow, expressed as a percentage. Since we have employed the term *stored energy* to represent the energy necessary to draw the bow (represented by the area under force-draw curve) we may write:

Formula 3)

Bow Efficiency = (E_k (arrow) / E_s (bow)) x100

This formula may be rewritten as:

E_k (arrow) = E_s (bow) x Efficiency / 100

Substituting in Formula 2, we have the
Formula 4)

E_s (bow) x Efficiency % / 100 $= W_a V^2 / 450240$

with all values as represented in previous definitions.

Sample:

Efficiency = (E_k / E_s) x100
Efficiency = (32.35 / 43.23) x100
Efficiency = 74.9 %
E_k = 32.35 ft.-lbs.
E_s = 43.23 ft.-lbs.

It is not too difficult to reason, and it is readily apparent when shooting a bow, that a given bow at a given draw length will not shoot heavy arrows fast as it will shoot light arrows. *But one fact that is not normally considered is that the* *same bow will impart more energy to the heavier arrow than it will to the light arrow.*

This occurs because the bow delivers greater percentage of the available stored energy to the heavier arrow, the less energy is left to be dissipated in the bow. Most archers can sense that a bow is quieter offers less recoil as arrow weight is increased — so no matter how insensitive bow hand and ear are, you can readily tell the difference by the jarring in your hand and the overall noise (not string slap).

Utilizing actual chronograph test data, the Formula 4) may be rewritten as follows:

Efficiency % =
$(W_a V^2 x100) / (450240 \times E_s) =$
$(484.75 \times 173.15^2 x100) / (450240 \times 43.23)$
Efficiency = 74.9 %

We can compute bow efficiency for different arrow weights and respective velocities and show that efficiency increases with arrow weight. This is true up to a point generally well beyond common practice in matching arrow weight to bow draw weight and draw length. Usually the peaking of the efficiency curve occurs at much heavy arrow weight than is ever practically shot from the bow because you encounter the problem of an extreme slow arrow.

We like to see arrow velocity and the efficiency tested, and presented, on the range of arrow weights encompassing variations of shaft size that typically might be shot from a given bow under field and hunting conditions. Usually this will cover a range of 100 or more grains and provide an excellent index of the bow's performance.

Most experimental data is subject to some, hopefully small, experimental error. To compensate for this, is a common practice to run duplicated tests and average the results. Depending upon the consistency of velocity readings, we will shoot a given arrow from a minimum of three times to as many as to 20 times to assure ourselves that the results are representative. If it is possible, we like to see arrow velocity readings fall within a maximum range of 1.5 feet per second before averaging the experimental value. In many circumstances, we have had repetitive and correlate within a total spread of velocity of 0.5 feet per second.

Another authoritative check on the validity of experimental velocity determinations is readily obtained by a plot of the experimental data for various arrow weights and the use of a curve-filter technique based on Dr. Paul Klopsteg concept of "virtual mass." This term is interchangeable with "effective mass" which is probably in wider use. The original introduction appeared in an article entitled "Physics of Bows and Arrows" by Paul E. Klopsteg, published in the *American Journal of Physics*, Volume 11, Number 4, August 1943, and it was repeated in a book he co-authored, entitled "Archery – The Technical Side."

The author defines "virtual mass" as «a mass, if it were moving with the speed of the arrow at the instant it latter leaves the string, would have precisely the kinetic energy of the limbs and the string at that instant.»

Dr. Klopsteg let "K" represent the virtual mass of a bow, and wrote:

$$rW = ½ (m + K)V^2$$

where

- W = energy required to draw the bow to full draw
- r = fraction of input energy remaining after deducting hysteresis loss
- rW = available energy
- m = mass of the arrow
- V = velocity of the arrow

He further determined, by testing a large number of bows, that Klopsteg virtual mass, was essentially a concern (as its designation might indicate) of a given bow.

Note that by using "available energy" Dr. Klopsteg avoided the inclusion of the hysteresis loss in the "virtual mass". Hysteresis in the self bows of that era was significant, and as near as I can determine, measured bow efficiencies were not always penalized by being based on the total work exerted to draw the bow. Instead, efficiency was calculated from a base value of total expended energy minus the amount of energy lost to hysteresis in the limb material. Hence it represented "net efficiency" rather than "overall efficiency."

In present practice, where modern materials and construction have greatly reduced hysteresis loss in conventional bows, it is more common to include the effect of hysteresis loss in the value of "virtual mass" and also to express efficiency as an "overall" value. This will account for the fact that excellent contemporary efficiency values lie in the mid-80 percentile range, while researchers like Hickman and Klopsteg mention efficiency values of 94 percent. The difference can probably be attributed to the disparity between "overall" and "net" efficiency.

If we rewrite the Klopsteg formula and include the hysteresis effect we have:

Formula 5)

$$E_s = ½ (m + K)V^2$$

where

Es = stored or potential energy (area under the force-draw curve) and the other values are as previously stated.

USE OF 'VIRTUAL MASS' VALUE

Now when we calculate "virtual mass" from chronograph data it may have a somewhat higher value, but its characteristic as a constant should not change, at least for conventional bows. I have found that for compound bows the values of virtual mass obtained from experimental data appear to have a wider variation from a mean value. This is probably due to variation in friction in. the compounding system from shot to shot and from arrow to arrow. This means that extension of arrow velocity data based on the virtual mass of the bow will be somewhat less dependable.

The principle use of the "virtual mass" value for a bow, other than direct comparison for evaluation, is in the extension of arrow weight-velocity curves for a given bow and for curve fitting when plotting experimental arrow weight-velocity data. We use it upon occasion for the first purpose, but we employ it for curve fitting for every arrow weight-velocity chart that is prepared.

The technique is based on the following manipulation of formulae (1) and (5):

1) $E_k = ½ \; m V^2$
5) $E_s = ½ (m + K)V^2$

rearranging Formula (5) we have:

$E_s = ½ \, m V^2 + ½ \, K V^2$

substituting from (1):

$E_s = E_k + ½ \, K V^2$

again from (1) we have:

$V^2 = 2 E_k / m$

substituting for V^2

$E_s = E_k + (K \times Ek) / m$

and

$(K \times E_k) / m = E_s – E_k$

Solving for K (the virtual mass)
Formula (6)

$K = m (E_s / E_k – 1)$

From our force-draw curve and chronograph tests all data is known and we can compute the virtual mass (K). In this instance, if we express the mass of the arrow in grains, the virtual mass will also be in grains since the conversion factor (from slugs to grains) will appear on both sides of the equation and cancel. Therefore…

$K \text{ (grains)} = W_k = W_a \text{ (grains)} (E_s / E_k – 1)$

E_s is the stored energy from the force-draw curve in foot-pounds.
E_k is the kinetic energy of the arrow in foot-pounds derived from the chronograph tests and Formula (1).

Sample:

$W_k = W_a (E_s / E_k – 1)$
$W_a = 454.5$
$E_s = 43.23$
$E_k = 33.03$
$W_k = 454.5 (43.23 / 33.03 – 1)$
$W_k = 454.5 (.309)$
$W_k = 140.44$ grains

Note: The average value of virtual mass calculated for feather fletching for this bow was 143.8 grains.

Now that we have determined the virtual mass for the bow – we use the average value calculated from all arrow weights and velocities as tested on the chronograph – it is possible to select any arrow weight we desire and compute the velocity for that arrow when shot from the test bow under the same conditions of test.

We use Formula (5) for this purpose and solve for V (the arrow velocity):

(5) $E_s = ½ (m + K) V^2$
$V^2 = 2 E_s / (m + K)$

Formula 7)

$V = \sqrt{2 E_s / (m + K)}$

Again, this is expressed in the absolute system. In order to use grain weights for "m" and "K" we must enter the conversion factors:

Formula 8)

$V = \sqrt{(2 \times 7000 \times 32.2 E_s) / (W_a + W_k)}$
$V = 671.0 \sqrt{E_s / (W_a + W_k)}$

V will be in feet per second
E_s will be in foot-pounds
W_a and W_k are in grains

To use Formula (8) for curve fitting when plotting experimental data it is only necessary to select convenient values of arrow weight at uniform increments over the approximate range of the test. With the virtual mass determined from the average of test data, the corresponding velocities are computed and plotted along with the strictly experimental points.

Sample:

$W_k = 143.8$ grains
$E_s = 43.23$

W_a	$W_a + W_k$	V
350	493.8	198.6
400	543.8	189.3
450	593.8	181.2
500	643.8	173.9
550	693.8	167.6

Example:

$V = 671.0 \sqrt{E_s / (W_a + W_k)}$
$V = 671.0 \sqrt{43.23 / 493.8}$
$V = 198.5$ ft. per second

CURVE AVERAGES OUT ERROR

The curve through the calculated velocity values will be an average arrow weight-velocity curve for the bow tested and should average out most individual experimental error.

The same technique is used for the plot of efficiency versus arrow weight using the uniform increments of arrow weight and the virtual mass as follows:

$$\text{Efficiency} = (E_s / E_k)\ x100$$

$$\text{Efficiency} = ((\tfrac{1}{2} m V^2) / (\tfrac{1}{2} (m + K) V^2))\ x100$$

Formula 9)

$$\text{Efficiency} = (m / (m + K))\ x100$$

Since the conversion factors cancel…

$$\text{Efficiency} = (W_a / (W_a + W_k))\ x100$$

where

W_a is the arrow weight in grains
W_k is the virtual mass in grains

Since these values are already available from the velocity calculations, it requires but a flip of the calculator to obtain the efficiency curve.

W_a	$W_a + W_k$	Efficiency
350	493.8	70.9%
400	543.8	73.6%
450	593.8	75.8%
500	643.8	77.7%
550	693.8	79.3%

Sample:

Efficiency = (350 / 493.8) x100
Efficiency = 70.9%

We believe that performance data derived in this manner is the most representative data possible, and for this reason the *Archery World* bow tests are prepared employing this technique. Certainly it requires more effort, but it tends to point up erroneous data so that it can be rechecked or eliminated. We feel that the end result is representative, comparative and credible.

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questo testo è stato trascritto includendo le correzioni presenti sulla fotocopia che Norb Mullaney ha inviato a Outlab nell'agosto 1989